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Parallel and serial processes in number-to-quantity conversion

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ABSTRACT

Converting a multi-digit number to quantity requires processing not only the digits but also the number's decimal structure, thus raising several issues. First, are all the digits processed in parallel, or serially from left to right? Second, given that the same digit at different places can represent different quantities (e.g., "2" can mean 2, 20, etc.), how is each digit assigned to its correct decimal role? We presented participants with two-digit numbers and asked them to point at the corresponding locations on a number line, while we recorded their pointing trajectory. Crucially, on some trials, the decade and unit digits did not appear simultaneously. When the decade digit was delayed, the decade effect on finger movement was delayed by the same amount. However, a lag in presenting the unit digit delayed the unit effect by 35 ms less than the lag duration, a pattern reminiscent of the psychological refractory period, indicating an idle time window of 35 ms in the units processing pathway. When a lag transiently caused a display of just one digit on screen, the unit effect increased and the decade effect decreased, suggesting errors in binding digits to decimal roles. We propose that a serial bottleneck is imposed by the creation of a syntactic frame for the multidigit number, a process launched by the leftmost digit. All other stages, including the binding of digits to decimal roles, quantification, and merging them into a whole-number quantity, appear to operate in parallel across digits, suggesting a remarkable degree of parallelism in expert readers.

1. Introduction

How do we combine the digits of a multi-digit number into a single quantity? In the visual system, the digits in a number such as "22" appear at distinct retinotopic locations and must therefore initially be processed independently, yet at some point they must be weighted according to their position in the overall number. The very same symbol, say 2, changes its meaning depending on its decimal role – e.g., it may mean two units (e.g. in "32") or twenty units (in "23"). The number recognition process must therefore be broken into several operations (Dotan & Dehaene, 2016): (1) Visual parsing and identification of each digit and their relative positions. (2) Binding each digit to a decimal role – units, decades, etc. (3) Quantifying each digit, i.e., multiplying the digit value by the weight implied by its decimal role, and merging these quantities. (4) Using the resulting quantity in further arithmetic or comprehension tasks.

The present study focuses on the 2nd and 3rd operations in this list: we aimed to understand how the digits of a multi-digit number are bound to decimal roles and how they are quantified. We examined these issues using a number-to-position mapping task: in each trial, participants were presented with a number and were asked to indicate the corresponding position on a number line. Quantities are thought to be internally organized along an internal continuum which has been likened to a "mental number line" (Berteletti et al., 2010; Cappelletti et al., 2005; Dehaene et al., 1993; Ruiz Fernández et al., 2011; Shaki et al., 2009; von Aster, 2000), so pointing at a physical number line is a good index of the internal quantity representation (Barth & Paladino, 2011; Booth & Siegler, 2006; Dehaene et al., 2008; Pinheiro-Chagas et al., 2017; Siegler & Booth, 2004; Siegler & Opfer, 2003). In our paradigm, the participants performed the task on a tablet computer while their finger position was continuously monitored. As we shall see, the finger trajectory provides remarkably detailed information about the cognitive processing stages underlying the digit-to-quantity conversion (Alonso-Diaz et al., 2018; Dotan et al., 2018; Finkbeiner & Friedman, 2011; Finkbeiner et al., 2008; Pinheiro-Chagas et al., 2017; Santens et al., 2011; Song & Nakayama, 2008a, 2008b, 2009; see a review in Dotan et al., 2019).

In past experiments, in which participants saw two-digit numbers

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and pointed at the corresponding location on a number line (Dotan & Dehaene, 2013, 2016), we analyzed how the decade and unit digits affected the participants' pointing – the finger horizontal position, or the direction to which the finger aims – in each time point during the trial. We found that for each time point throughout the trial, the effect of the decade digit on the finger position/direction was larger by a factor of 10 than the effect of the unit digit, as expected given their decimal roles. Consequently, we suggested that the decade and unit digits were quantified simultaneously. In fact, however, several cognitive architectures may underlie this finding. These architectures differ from each other in the view they take with respect to three issues:

(1) Holistic versus decomposed processing. Some researchers assume that the entire two-digit string is recognized holistically, similar to the lexical route for the visual recognition of known words (Coltheart et al., 2001; Ellis & Young, 1996; Friedmann & Coltheart, 2018). This would require that all 2-digit numbers and their corresponding quantities be lexically stored (for counter-evidence, see Cappelletti et al., 2008; Cohen et al., 1994). In contrast, others assume that the two digits are processed in a decomposed manner, i.e., each digit is identified separately, and the quantity is computed separately for each digit (Moeller et al., 2009; Nuerk & Willmes, 2005).
(2) Serial versus parallel processing. If the two digits are processed separately (in decomposed manner), they may be processed in parallel or serially (seriality is usually assumed to be from left to right). Parallel-processing models usually assume that each digit is processed independently of the other, and this independence allows to process the two digits in parallel. Serial processing models usually assume some kind of dependency between the mechanisms that process the two digits – e.g., that one digit can only be processed after the other – and this dependency creates seriality. Previous studies have concluded that the digits are processed in parallel in 2-digit and 3-digit numbers, but serially in longer numbers (Bahnmueller et al., 2016; Dotan, Eliahou, & Cohen, submitted for publication; Friedmann et al., 2010; Meyerhoff et al., 2012).
(3) Single-decision versus cascaded decisions. Our task required the participants not only to understand the quantity represented by the two-digit number, but also to direct their finger according to this quantity. The transition from quantity-representation to finger-direction can be made in a single decision per trial, after which the finger direction is no longer changed (Dotan & Dehaene, 2016), or alternatively in a series of decisions, each of which modifies the finger direction (Alonso-Diaz et al., 2018; Chapman et al., 2010; Dotan et al., 2018; Erb et al., 2016; Friedman et al., 2013; Pinheiro-Chagas et al., 2017).

Our previous findings, that the decades and units affect the finger movement simultaneously and in 10:1 ratio (Dotan & Dehaene, 2013, 2016), can be explained by several models, each of which takes a different view with respect to the 3 issues above. One possibility, termed the *lexical model*, is that the entire two-digit string is recognized holistically. Because the model assumes only one combined holistic quantity, the two digits would affect the finger movement in 10:1 ratio irrespectively of whether we assume a single decision or cascaded decisions. A second possibility is the *parallel-decomposed model*: the digits are processed in a decomposed manner but in parallel, again either with a single decision per trial or with cascaded decisions. A third possibility is the max *model*: it assumes that the decade and unit digits are quantified separately, either serially or in parallel, but on each trial there is only a single decision to deviate the finger, towards the merged decade + unit quantity, and the decision is made only after both per-digit quantities were fully computed. Even if one digit was processed before the other, it would wait for the quantification of the other digit to be completed too, so the decade and unit quantities would still affect the finger movement in parallel, i.e., in 10:1 ratio throughout the trial.

To evaluate these possibilities, we asked participants to perform the trajectory-tracked number-to-position task with two-digit numbers. We computed how the decade and unit quantities affected the finger direction in each time point during the trial. Crucially, we systematically varied the temporal order and delay separating the onsets of the decade and unit digits: on some trials the decades appeared shortly before the units, or vice-versa. This method allows examining the dependencies that link the two digits. If all four operations described above (parsing, binding to decimal role, quantification, movement) unfold independently for each digit (parallel-decomposed model), delaying a digit by a delay Δt should delay the effect of this digit on finger movement by the same Δt, but it should not delay the effect of the other digit on finger movement. Conversely, if there is full dependency between the digits – e.g., because all digits are needed to recognize the full number as a lexical item (lexical model), or because the decision to move the finger depends on the availability of all digits (max model) – then delaying either digit by Δt would delay the effects of both digits on the finger movement by Δt. Note that there are other possibilities. For instance, the dependency between digits does not have to be symmetric: it is possible that the processing of the unit digit depends on the decade digit but not vice versa. Such would be the case if we assume purely sequential processing – first the decade digit, then the unit digit.

Delaying the onset of a digit might also perturb the binding of digits to decimal roles. If the decade digit transiently appears alone on screen, it might be incorrectly quantified as a single-digit number. Conversely, a stand-alone unit digit might be transiently misinterpreted as the decade of a two-digit number. To reduce the likelihood of such binding errors, in our experiments the digits always appeared in predictable locations on screen, and when delaying the onset of a digit, its position was temporarily occupied by a placeholder character (i.e., only two-character strings were always displayed). Nevertheless, we shall see that there was still evidence of binding errors.

We performed three experiments that addressed these issues. Experiment 1 presented two-digit numbers, and the unit digit appeared either simultaneously with the decade digit, or after a variable delay. Experiment 2 extended the method to delay either the unit digit or the decade digit (in different trials). Experiment 3 extended the conclusions of the first two experiments to 3-digit numbers.

2. General method

2.1. Participants

All participants were right-handed adults with no reported cognitive disorders, gave informed consent, and were compensated for participation. Their mother tongue was Hebrew, in which words are written from right to left but numbers are written like in English. Hebrew and left-to-right readers were found to exhibit similar patterns of results in our paradigm (Dotan & Dehaene, 2013).

2.2. Procedure

On each trial, the participants saw a two-digit number on an iPad tablet computer (screen resolution: 1024 × 768 px) and pointed at the corresponding position on a number line. A horizontal number line,

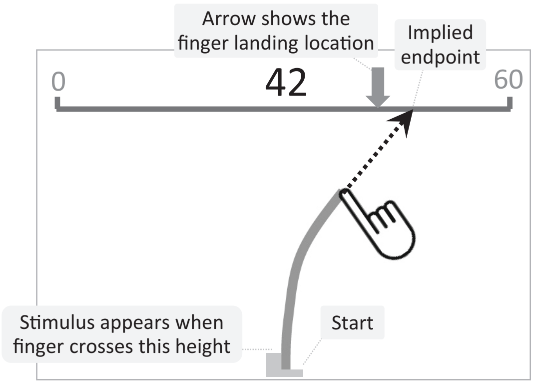

Fig. 1. Number-to-position mapping with finger tracking – screen layout. Participants pointed to the location of 2-digit numbers on a horizontal number line. On each trial, they dragged their finger from a fixed starting point to the number line.

whose only marks were numeric labels at its ends, occupied the top of the screen throughout the experiment (Fig. 1). Touching the starting point made a fixation indicator appear above the middle of the number line. Finger movement (crossing y = 50 pixels from the bottom of screen) caused the fixation indicator to be replaced by the target, which was presented for a limited time (the specific target is described below for each experiment, but it always included the 2-digit number). The participants moved their finger to what they judged to be the corresponding position on the number line, and an acknowledgement arrow appeared at the position marked by the finger. Trials were invalidated by lifting the finger in mid-trial, moving backwards, starting a trial with sideways (rather than upward) movement, or moving too slowly: except a grace period of a trial's first 300 ms, the finger had to reach the number line within 2 s and y = 30% within 1 s, with linear interpolation; and to keep momentary speed of at least 6 mm/s. Invalid trials were excluded from analysis, and the target reappeared later in the experiment. Trials with movement time < 200 ms were excluded post-hoc. The experimental protocol was approved by the Tel Aviv University ethics committee.

Our experiment ran on a proprietary iPad software. Recently we developed an equivalent freeware that runs with Python on desktops. This freeware, together with a matlab-based framework that we developed to analyze trajectory data, are available on http://trajtracker.com. The raw data is provided as supplemental online material.

2.3. Data preprocessing

The finger position was sampled at 60 Hz, recorded as a sequence of time stamped x and y coordinates, and transformed into a fixed sampling rate of 100 Hz using cubic spline interpolation. The x,y coordinates were then separately smoothed with Gaussian weighting ($\sigma = 20$ ms). For each time point in the finger trajectory, θ_t was defined as the direction between the finger coordinates at times t - 10 ms and t, and the *implied endpoint* (iEP) was defined as the position on the number line that the finger would reach if it keeps moving in direction θ_t. Implied endpoints were cropped so not to exceed each end of the number line by more than 5% its length, and were undefined for sideways movement ($|\theta_t| > 80°$). The trial *endpoint* is the position where the finger crossed the number line.

2.4. Statistical analysis

2.4.1. Factors affecting the finger movement

Our main analysis method of finger trajectories aimed to identify the factors that affect the finger movement in different times during the trial (Dotan et al., 2019). The trajectory data – implied endpoints – was analyzed on each time point, starting from $t = 0$ (when the finger left the "start" point), in 50 ms intervals. For each time point, the implied endpoints of all trials were regressed against four predictors: the decade (0, 10, 20, …, denoted D), the unit digit (U), the target of the previous trial (N-1), and a bias function (defined in Eq. (1) for a 0–60 number line). The bias function (SRP, which stands for "Spatial Reference Points") is assumed to reflect a spatial aiming strategy that relies on the middle and ends of the number line as anchor points (Barth & Paladino, 2011; Dotan & Dehaene, 2013; Rouder & Geary, 2014; Slusser et al., 2013; such anchor points may be used even by young children, Feldman et al., 2019; Huttenlocher et al., 1994). The idea is that to determine the number-line location corresponding with a particular quantity, the participant estimates the ratio between the target location's distances to the two nearest anchor points, and these distances are estimated using a logarithmic scale[1]:

$$\mathrm{SRP(N)} = \begin{cases} 30 * \dfrac{\log(N+1)}{\log(N+1) + \log(31-N)} - N \text{ For N} \leq 30 \\ 30 + 30 * \dfrac{\log(N-30+1)}{\log(N-30+1) + \log(31-N)} - N \text{ For N} \geq 30 \end{cases} \tag{1}$$

One regression was run for each participant and time point. The regression coefficients (b) were averaged over participants for each predictor in each time point. These average coefficients are denoted in the text as b[U] (unit effect), b[D] (decade effect), b[N-1] (previous target effect) and b[SRP]. When plotting these results, we show the average b value for each regression predictor as a function of time (e.g., Fig. 2b). In these plots, each point is the average b value (over participants) of one predictor in one time point. Each vertical "slice" reflects the results from the regressions of all participants in a single time point, i.e., the relative weights of the different predictors in that time point are visualized as the relative heights of the respective per-predictor lines. Each line in these plots reflects the buildup of one predictor's effect over time, as the line presents the predictor's regression coefficients (averaged over participants) in each time point. For example, in Fig. 2b we can see that the decade and unit digits started affecting the finger movement about 400 ms after the trial started: the regression lines of b[D] and b[U] start rising at that time.

To examine whether the effect of a given predictor in a specific time point was significant across participants, the per-participant regression b values of that predictor in that time point (significant and non-significant) were compared with zero using *t*-test (one-tailed *p*, because we predicted only positive b values). The plots use full dots to denote b values significantly higher than 0. In Fig. 2b, 400 ms was the first time point in which both the decade digit and the unit digit significantly affected the finger movement.

In previous studies with the trajectory-tracked number-to-position paradigm, we used a logarithmic predictor to account for potential logarithmic quantity representation. The log predictor was not used here, because we recently showed that it captures a temporal bias rather than logarithmic representation (Dotan & Dehaene, 2016), but including it yielded essentially the same results.

2.4.2. Comparing regression effects

Regression effects were compared in two kinds of situations. Some analyses examined the difference between the effects of two predictors in the same experimental condition (e.g., the decade effect versus the unit effect). Other analyses compared the effect of a single predictor between two different experimental conditions, to examine how a certain experimental manipulation delayed this predictor (e.g., if we

[1] The spatial aiming strategy maps each number to the range 0–60. Here, we defined the regression predictor SRP(N) as the difference between this mapped value and the value of N. This was done in order to prevent high correlation between the regression predictors N and SRP(N).

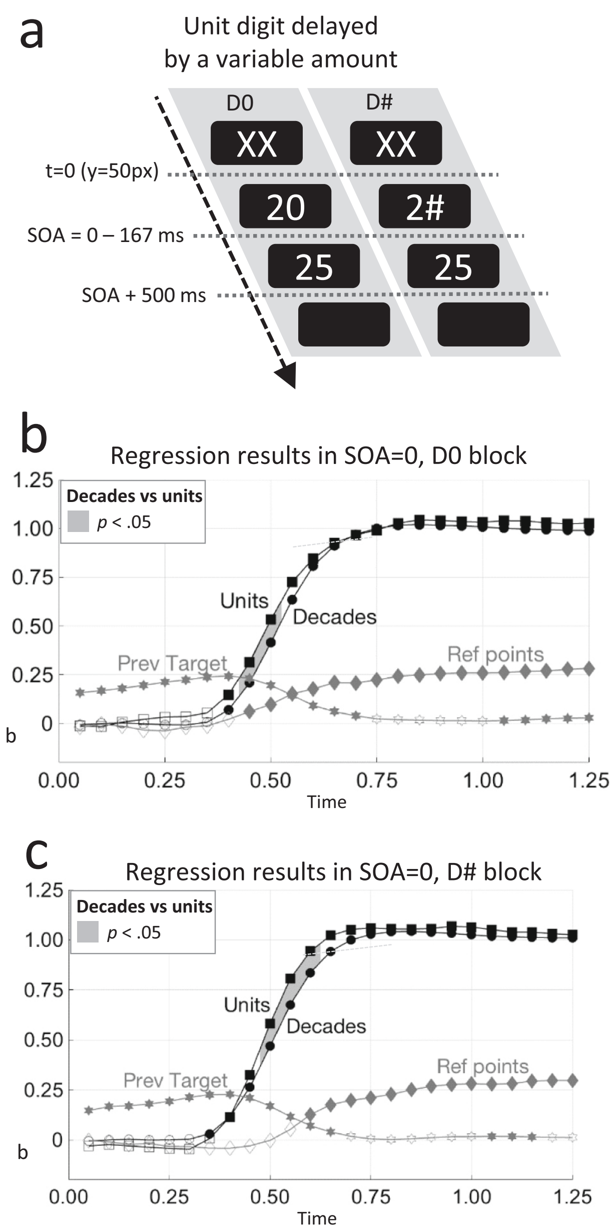

Fig. 2. Experiment 1 design and baseline results. (a) Task design: the fixation was XX. At $t = 0$, the decade digit appeared and the unit position was occupied by a '0' or '#' character (in two separate blocks). The placeholder character was replaced by the unit digit after 0, 33, 67, 100, 133, or 167 ms. The '0' placeholder makes the transient stimulus a valid two-digit number, and was aimed to reduce errors in binding of digits to their decimal roles. (b,c) Time course of the factors that affected the finger movement in the SOA = 0 condition (simultaneous presentation of the two digits). For each participant, time point, and condition, the finger direction (implied endpoint) was regressed against the decade (0, 10, 20, etc.), the unit digit, the target of the previous trial, and a bias function. The regression coefficients were averaged over participants and plotted as a function of time. Full dotes denote coefficients significantly larger than 0 (*t*-test). The decade and unit effects rise together in a ratio of nearly 1:10, with a small over-weighting of the units (grey area = significant difference). Note that the decade and unit regression predictors were scaled in 10:1 ratio, so identical b values for decades and units mean that the decade and unit quantities were processed in 10:1 ratio.

delay the unit digit by 100 ms, would its effect on finger movement be delayed by the same amount?). Note that in all our regression analyses, the unit predictor values were 0, 1, 2, … 9 whereas the decade predictor values were 0, 10, 20, … 90, so identical values of the decade and unit regression coefficients (b[D] and b[U]), i.e., overlapping curves in the regression plots, mean that the unit and decade quantities were processed in exactly 1:10 ratio, in accord with their decimal roles.

2.4.2.1. Interpreting a discrepancy between regression effects. In the regression plots, the difference between two regression effects is visualized as a discrepancy between the corresponding regression curves (e.g., the grey area in Fig. 2b). Such discrepancy can be interpreted in two ways. One explanation is in terms of the amplitude of the regression effects: in Fig. 2b, this would mean that the curve of the decade effect is *lower* than the unit curve, reflecting that the decade predictor had a smaller weight in the regression model than the unit predictor. Such explanation means that the decade digit affected the finger movement *less* than the unit digit (i.e., less than the expected 1:10 ratio). Another explanation is that the decade curve is *delayed* with respect to the unit curve – namely, the two effects had the same amplitude (the digits were processed in exactly 1:10 ratio), but the finger was first affected by the unit digit, and the effect of the decade digit kicked in a little later. Corresponding with these two interpretations, we used two methods to compare regression effects.

2.4.2.2. Comparing the amplitudes of two effects in each time point. The first method examined whether one effect was significantly larger than another: for each time point, the per-participant b values were compared between the two conditions using paired *t*-test (one-tailed *p* is reported, as we always had a clear prediction as for which effect should be larger). This comparison was restricted to the time window when the regression curves were on the rise (this was always in the range 350–650 ms), because these time windows are the most informative (e.g., see the area shaded in grey in Fig. 2b).

2.4.2.3. Estimating the delay between two factors. A second method quantified the *delay* between two regression effects. Intuitively, quantifying this delay amounts to measuring the horizontal distance between the two regression curves, and this could be done by shifting one curve horizontally until it reaches a best overlap with the other curve. Mathematically, this delay was defined, based on the average b values of the two regression curves, as $\min_{\Delta t}\int(b_{cond1}(t) - b_{cond2}(t-\Delta t))^2 dt$, i.e., the Δt that minimized the sum of squares of the vertical distances between the two curves. Here too, the integral was restricted to the most informative time window, i.e., when the regression curve was on rise (we verified that the results were robust to this choice and were replicated for several time windows). To approximate the integral, the regressions were run in 10-ms intervals and the b values were interpolated to 1-ms granularity with cubic spline interpolation.

2.4.2.4. Calibrating the effect sizes by their asymptotes. In our analysis of delay between regression effects, a difference in the amplitudes of two effects may confound with their estimated delay (Section 2.4.2.1): if one effect is smaller than another, it would artificially appear as delayed. The reason is that in our trajectory tracking paradigm, the regression analyses cannot really distinguish between a *reduced effect size* and a *delayed effect*. One way to address this problem is to normalize the regression values by dividing each b curve by its endpoint value (intuitively, this means vertically "stretching" the two curves until they converge to a common asymptote). This rescaling method makes two assumptions. First, it assumes that a pure delay is a transient effect, i.e., the effect of a digit on finger movement may take longer to build up, but eventually it would reach the full effect size (our regression curves suggest that this is indeed the case). The end-of-trial values of the regression predictors therefore reflect their relative amplitudes irrespectively of any delays, and scaling according to these amplitudes would control for their effect. Second, our rescaling method assumes that the downsizing of a regression effect A relative to

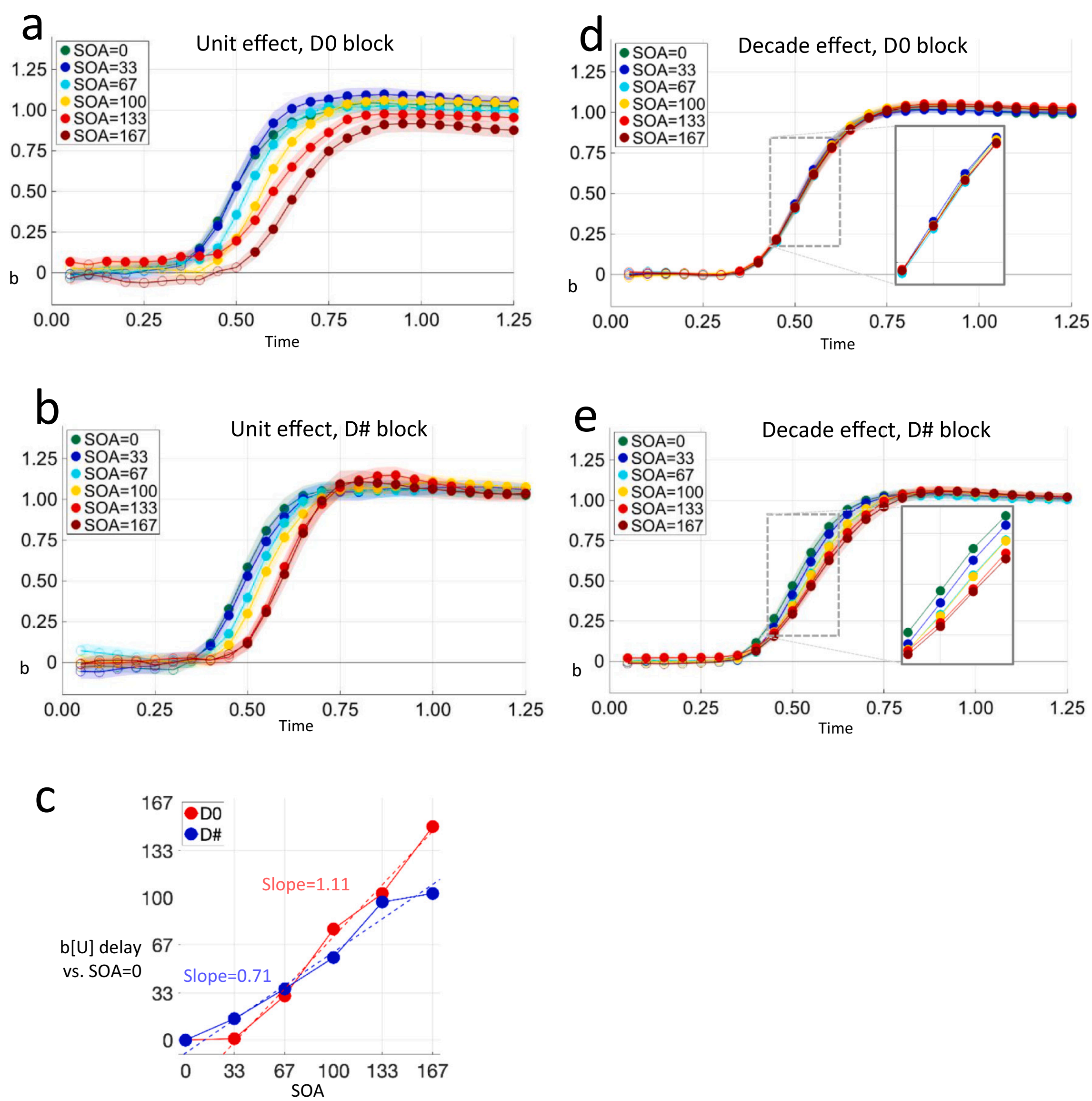

Fig. 3. Time course of the decade and unit regression effects in Experiment 1. Each SOA condition is plotted in a different color (same plot type as panel c). The shaded area shows one standard error above/below mean. (a) The unit effect b[U] showed an IDLE pattern in the D0 block: delaying the unit digit by 33 ms had almost no effect on b[U], whereas longer delays caused a linearly-increasing delay of b[U]. (b) In the D# block, b[U] did not show the IDLE pattern. (c,d) Increasing the SOA created a small decrease/delay in the decade effect in the D# block but not in the D0 block. (e) Influence of the unit digit at various SOAs. In the D0 condition, the shortest SOA (33 ms) had no effect on b[U], and extending the SOA beyond that created a linearly increasing delay (IDLE pattern). The dashed lines are regressions of the b[U] delay against SOA (SOA = 0 excluded). In the D0 block, this regression has a slope that approached 1.0 and it crosses the x axis at SOA = 35 ms, i.e., extending the SOA beyond 35 ms delayed the unit effect by SOA-35 ms.

effect B remains constant throughout the trial.

This rescaling method was applied only when there was a theoretical reason to assume that the regression effects were increased in some conditions relatively to other conditions. We verified that the results were not an artifact of the rescaling method: in all analyses in which rescaling was used, the results were essentially the same with and without rescaling.

2.4.3. ANOVA

Some analyses in this study were ANOVA whose exact design is detailed in the text below. Our goal in this study was not to explain inter-individual differences, but to focus on the within-participant factors that affect people's behavior in the number-to-position task. We therefore used only repeated measures ANOVA and we report effect sizes as partial η^2, a measure independent of the between-participant variance. To provide the full picture, we also report η^2 for one-way ANOVAs, and generalized η^2 (Bakeman, 2005; Olejnik & Algina, 2003), denoted $\eta_G{}^2$, for ANOVA with several factors.

3. Experiment 1: delaying the unit digit

In this experiment, the decade digit always appeared at $t = 0$ and the unit digit appeared either simultaneously or after a short delay. If the processing of the unit digit unfolds independently of the decade digit, any delay in the unit digit onset should be fully reflected in its effect on the finger movement.

To discourage the binding of digits to incorrect decimal roles, the position of the delayed unit digit was temporarily occupied by a '0' placeholder character (Fig. 2a, "D0" block) – e.g., the target number 25 appeared as 20 and then changed to 25. With a '0' placeholder, the transient stimulus (20) is a valid two-digit number. We hoped this would facilitate the processing of the displayed decade digit as part of a two-digit number (rather than as a single-digit number), and thus discourage binding to incorrect decimal roles. Still, a possible disadvantage of the ‘0’ placeholder is that, being an acceptable digit, the ‘0’ may undergo the full processing pathway on top of the target unit digit or instead of it. As a result, the finger movement would reflect a mixture of the target unit digit and the ‘0’ placeholder digit, and this would reduce the average unit effect on finger movement. To control for this possible confound, we added a second block with a non-digit placeholder character (D# block). The ‘#’ character should not undergo the digit processing pathway, and should not cause the unit under-representation artifact, perhaps at the cost of more errors in binding digits to decimal roles. By comparing the D0 and D# blocks we could examine the putative presence of binding errors.

3.1. Method

28 right-handed adults, aged 25; 10 ± 2; 7, performed two blocks of the number-to-position task. The transient unit placeholder character was ‘0’ in one block and ‘#’ in the other. We used a 0–60 number line (note that a long number line provides a larger range of numbers, whereas a short number line provides better measurement precision). In both blocks, the decade digit always appeared at $t = 0$, and the unit digit appeared at t = 0, 33 ms, 67 ms, 100 ms, 133 ms, or 167 ms (mixed design; Fig. 2a). Each target number between 10 and 50 was presented twice per block and SOA (492 trials per block).

The fixation indicator was uppercase XX. In the SOA = 0 condition, the XX changed into the target number at $t = 0$. In longer SOAs, the decade digit appeared at t = 0, whereas the unit position changed at t = 0 into ‘#’ or ‘0’, and after the SOA duration – to the target digit. Both digits disappeared 500 ms after the unit digit onset. Note that at t = 0, a visual change occurred in both decimal positions, in order to minimize the between-digit differences in attentional, alerting or spatial bias.

The finger trajectory data of each SOA condition was separately submitted to the regression analysis and analyzed for between-condition delays as described in Section 2.4. In the D0 block, we suspected that b[U] may not converge to the same value in all SOA conditions (due to an effect of SOA on the amount of processing the ‘0’ placeholder). As we shall see below, this was indeed the case. Thus, the delay estimations were not based on the raw b values but on the ratio between b[U] and its endpoint value, as explained in Section 2.4.2.4.

3.2. Results

3.2.1. General analyses

The end-of-trial-time measures and the trajectory regressions (see Appendix) replicated the main patterns observed for our number-to-position paradigm in previous studies (Dotan & Dehaene, 2013). Most importantly, in both blocks, when the decade and unit digits were presented simultaneously (SOA = 0), the decade and unit digits affected the finger movement in ~10:1 ratio throughout the trial: the regressions showed almost-overlapping effects of b[D] and b[U], with only a small over-weighting of b[U] relative to b[D] during a transient time window (Fig. 2b,c).

3.2.2. Partial processing of the 0 placeholder

The trajectory data of each SOA condition was submitted to the two-stage regression analysis as described in Section 2.4.1. In the D0 block, the influence of the units digit (the regression coefficient b[U]) showed lower values as the SOA increased (Fig. 3a), and importantly, this trend was observed even in the trajectory endpoints. Namely, at the end of the trial, the unit effect b[U] was smaller for larger SOAs. This suggests that, as we predicted, the ‘0’ placeholder character was partially processed as the target unit digit, thereby reducing the observed unit effect, and that the degree of this partial processing increased with the duration of presenting the ‘0’ placeholder. The dependency of this effect on SOA was confirmed by a repeated measures ANOVA on the b[U] values at the trial endpoint, with SOA as a single within-subject numeric factor ($F(1,27) = 13.6$, $p < .001$, $\eta^2_P = 0.34$, $\eta^2_G = 0.09$).

This conclusion, that the SOA effect on b[U] resulted from partial processing of the ‘0’ placeholder, is strengthened by the finding that the effect was observed only in the D0 block – no corresponding linear trend in b[U] was found in the D# control block ($F(1,27) < 0.2$). The difference between the blocks was significant: a two-way repeated measures ANOVA on b[U](endpoint), with SOA as a within-subject numeric factor and the block (D# or D0) as a within-subject factor, showed a significant SOA × Block interaction ($F(1, 27) = 7.83$, $p = .01$, $\eta^2_P = 0.22$. $\eta^2_G = 0.02$).

3.2.3. D0 block: the effect of delaying the unit digit

We now turn to the main goal of this experiment. If the unit digit is processed independently of the decade digit, then delaying the unit digit onset (extending the SOA) should delay the effect of the unit digit on finger movement (b[U]) by the same amount. Fig. 3a clearly shows that this was not the case: the shortest SOA (33 ms, blue line) had virtually no impact on b[U] – i.e., when the unit digit onset was delayed by 33 ms, it had the same effect on finger movement as in the no-delay condition (SOA = 0, green line). Only extending the decade-unit SOA beyond 33 ms created an increasingly larger delay in b[U]. To quantify this visual impression, we computed the delay between the unit effect b[U] in each SOA versus SOA = 0, using the method described in Section 2.4.2. As Fig. 3c (red line) shows, delaying the onset of the unit digit by 33 ms did not delay at all the unit effect on finger movement (for SOA = 33 ms, the b[U] delay is 0). Delaying the unit onset by more than 33 ms increased the b[U] delay, roughly as a linear function of the SOA. To show that this “step” in the delay-by-SOA graph is significant, we showed that the delay-by-SOA graph forms a linear function only when excluding SOA = 0: when regressing the b[U] delay on SOA, including only SOAs > 0 (dashed red line in Fig. 3c), the residual of SOA = 0 was significantly higher than the residuals of the other SOAs (Crawford and Garthwaite's (2002) $t(4) = 6.66$, two-tailed $p = .003$) – i.e., the delay of SOA = 0 significantly deviated from the regression line. In contrast, the delays of other SOAs were not deviants: repeating the same leave-one-out procedure for each SOA showed no significant deviation from the regression line for any SOA except 0 (Crawford and Garthwaite's (2002) $|t(4)| < 1.68$, two-tailed $p \geq .17$).

To quantify by how much the SOA delayed the unit digit, we examined the slope of the delay-by-SOA regression (excluding SOA = 0; dashed red line in Fig. 3c). The regression slope was close to 1 (slope = 1.11, $r^2 = 0.992$) – i.e., beyond SOA = 33 ms, extending the SOA by a certain amount delayed the unit effect b[U] by a similar amount.

The delay values reported here were computed based on the regression coefficients (b[U] values) in the time window 450–460 ms, after rescaling the b[U] values by their asymptote (endpoint values) as explained in Section 2.4.2.4. We verified that the results were essentially the same when using the raw b[U] values (without rescaling), as well as when computing the b[U] delays based on wider time windows (350–750 and 350–1000 ms).

How can we explain the step function in the delay-by-SOA graph – i.e., why did the SOA start delaying the unit effect b[U] only for SOA > 33 ms? One explanation is that the unit quantity was not processed as soon as the unit digit appeared, but only after a short interval – an idle time window in the units processing pathway. During this idle time window, the identity of the unit digit is not yet needed. A small delay (33 ms) in the onset of the unit digit is fully absorbed in this idle time window and has no impact on finger movement, and larger delays are partially absorbed in the idle time window. This pattern of results – b[U] delay that increases linearly with SOA, starting from a certain threshold SOA – is hereby referred to as the “Idle Digit Latency Effect” pattern (or IDLE pattern in short).

The duration of the idle time window in the units processing pathway should be identical with the smallest SOA that delays the unit quantification and consequently delays the unit effect b[U]. Based on our data, we estimated the duration of the idle time window as 35 ms – the SOA for which the delay-per-SOA regression (dashed red line in Fig. 3c) predicts delay = 0.

3.2.4. D# block: no IDLE pattern in the unit effect

The results in the control block did not show the IDLE pattern: the b [U] delay increased more or less linearly with SOA, starting from the smallest SOA (Fig. 3b and the blue line in Fig. 3c). The delays were computed based on the raw regression coefficients in the time window 450–650 ms (very similar delays were obtained with wider time windows: 350–750 and 350–1000 ms).

As in the D0 block, we regressed the b[U] delay by SOA (dashed blue line in Fig. 3c). The slope of this regression was 0.71 ($r^2 = 0.96$), i.e., when the unit digit onset was delayed by a certain amount, the unit effect on finger movement was delayed by only 71% of that amount. Why was the unit-onset delay reflected only partially in the finger movement? One explanation is that delaying the unit onset actually delayed in the unit effect on finger movement to a similar extent, but another factor partially masked this delay. We propose that this other factor is errors in binding the digits to their decimal roles, as explained next.

3.2.5. Errors in binding digits to decimal roles

Binding a digit to an incorrect decimal role affects the digit's quantification: if a unit digit is bound to the decade role, it would be processed with 10 times its real quantity; and if a decade is bound to the unit role, it would be processed as its quantity. In our regression analysis, binding errors would decrease the decade effect b[D] and increase the unit effect b[U]. If the binding error is transient – i.e. there is some confusion in the beginning of the trial, which is corrected later in the trial – the decade and unit quantities would be biased only during a transient time window.

We predicted that binding errors would be more frequent in the D# block than in the D0 block, because displaying the decade digit alone, with no other digit on screen, as is the case during the SOA period in the D# block (but not in the D0 block), may increase the likelihood of binding errors. Moreover, if the decade digit is displayed alone on screen for a longer duration, the likelihood of binding errors should increase – i.e., in the D# block, binding errors should be more frequent for larger SOAs. In contrast, in the D0 block the decade digit never appeared alone on screen, so increasing the SOA should not increase the likelihood of binding errors.

To evaluate the binding-errors hypothesis described above, we examined its prediction that larger SOA would decrease the decade effect b[D] in the D# block (but not in the D0 block). We analyzed b[D] by SOA using repeated measures ANOVA (one ANOVA for each time point), with b[D] as the dependent variable, SOA as a numeric within-subject factor, and the participant as the random factor. Assuming that many binding errors are transient and are eventually corrected, the SOA should affect b[D] only in an intermediate and relatively early time window. In the D# block, this was indeed the case: b[D] decreased as SOA increased, and this effect was observed only in an early time window, during the b[D] buildup (400–750 ms, $F(1,27) > 5.64$, $p < .03$, $0.17 < \eta^2_P < 0.65$, $0.02 < \eta^2_G \leq 0.10$; the linear trend is clearly visible in the inset in Fig. 3e). In contrast, as predicted, this pattern was not found in the D0 block in any time point (Fig. 3d, $F(1,27) < 2.3$, $p > .14$).

These results indicate that when the placeholder character was '#', the decade digit was sometimes bound to the unit role, and that longer SOAs increased the probability of such an erroneous binding. Using the '0' character as placeholder largely eliminated these binding errors.

3.2.6. How binding errors affect the analysis of delays

One reason that binding errors are important is that they may bias our analysis of the timing of the unit effect b[U], and particularly – our estimation of how delaying the unit digit onset delays its effect on finger movement. In particular, binding errors may explain why the delay-by-SOA graph (Fig. 3c) showed the expected slope of about 1.0 only in the D0 block, and showed a smaller slope in the D# block.

The idea is simple: as confirmed by the analyses above (Section 3.2.5), binding errors cause the decade digit to be processed by less than its real value, and the unit digit by more than its real value – i.e., binding errors decrease the decade effect b[D] and increase the unit effect b[U]. This may bias the analysis of delays: the analysis of delays does not distinguish between a *larger* b[U] effect and an *earlier* b[U] effect (see Section 2.4.2.3), so the elevated unit effect b[U] (resulting from binding errors) would appear, in the delay analysis, as an earlier b [U] effect – i.e., the b[U] delay would be under-estimated. If the likelihood of binding errors increases with SOA (as in the D# block), the delay analysis would also under-estimate the effect of SOA on the b[U] delay. We shall now explain this mechanism in detail.

The key point is that the bias in our temporal analysis of the unit effect b[U] – the amount by which b[U] appears to be earlier than it really is – become larger when the likelihood for binding errors is higher, i.e., when the SOA is larger. This has two consequences. First, it would bias the b[U] delay estimation for any particular SOA. When estimating the delay between SOA = 0 and any other particular SOA (say, SOA = 100), the latter SOA induces a larger temporal bias in the unit effect b[U] (due to the larger likelihood of binding errors). Namely, binding errors may make b[U] appear too early even in SOA = 0, but they still do so to a larger extent in SOA = 100. As a result, the delay analysis under-estimates the b[U] delay between SOA = 0 and SOA = 100. We can think of the analysis of delays as reflecting the sum of two factors: a genuine delay in the unit effect b[U], induced by delaying the onset of the unit digit (this is the effect that we wanted to measure in Sections 3.2.3 and 3.2.4); and an apparent negative delay, which originates in the effect of SOA on binding errors.

The second consequence occurs because the size of the apparent negative delay of a particular SOA (i.e., the degree of under-estimation of the b[U] delay) depends on the difference between that SOA and SOA = 0 in the likelihoods of binding errors. Crucially, in the D# block this Δlikelihood increases with SOA. Namely, not only does the delay analysis under-estimate the b[U] delay, but the degree of this under-estimation increases with SOA. As a result, in the D# block, the analysis under-estimates the effect of SOA on b[U] delay. In Fig. 3c, this under-estimation is reflected as the slope of the D# curve (blue) being lower than the expected 1.0.

If we could measure the genuine b[U] delay, we may have observed a delay-by-SOA slope of 1.0, but the b[U] delay cannot be directly measured because it is partially masked by the apparent negative delay resulting from binding errors. Crucially, these biases are specific to the D# block. As we saw above, in the D0 block the SOA manipulation does not induce binding errors, so the analysis of delay-by-SOA is not hampered. Thus, the delays computed for the D0 block should be taken as more reliable.

3.3. Discussion of experiment 1

Experiment 1 presented two-digit numbers in which the unit digit was delayed by a variable amount. In the critical block (D0), in which the transient stimulus was a valid two-digit number, a clear IDLE pattern was observed: a short delay in the unit digit onset (33 ms) did not delay the unit effect on finger movement, and left the unit effect virtually identical with the no-delay condition; whereas longer delays caused a linearly-increasing delay in the unit effect. This pattern suggests the existence of an idle time period in the units processing pathway, during which the unit digit is not yet needed, so its absence has no impact on the number-to-quantity conversion process. Small delays in the onset of the unit digit (SOA ≤ T_{idle}, where T_{idle} denotes the idle time window duration) are fully absorbed in this idle time window, so they have no effect on the finger movement. Larger delays (SOA > T_{idle}) are partially absorbed, so the unit effect on finger movement is delayed by SOA - T_{idle}. The results of Experiment 1 suggest that $T_{idle} \approx 35$ ms.

This idle time window resembles the classical psychological refactory period effect (PRP) (Pashler, 1984, 1994; Sigman & Dehaene, 2005), in which the processing of task A is delayed until the processing of a simultaneous task B is completed. In our case, the unit processing may wait for the completion of a process triggered by the decade digit: the process is a bottleneck, and until completed, the unit processing cannot continue and must wait, hence the idle time window. While PRP effects typically suggest serial processing, note that our findings are also quite unlike the classical PRP effects in a specific respect: here, as in our previous experiments with 2-digit numbers (Dotan & Dehaene, 2013, 2016), in the condition where all information appears simultaneously (SOA = 0), the decade and unit effects did not have a sequential impact on finger movement, but a parallel impact. We revisit this issue in Experiment 3.

Another finding was that the decade effect b[D] decreased with SOA in the D# block. This pattern suggests that the decades and units were sometimes confused, so that decades were processed as units (and perhaps also vice versa). One reason for this to occur could be that the digits were sometimes bound to an incorrect decimal role. The likelihood of these binding errors was increased by displaying the decade digit alone on screen for increasing durations, and they were nearly eliminated when only valid two-digit numbers were presented (as in the D0 block). Still, some degree of binding errors may exist even when both digits are presented simultaneously, which may explain why our number-to-position paradigm, even in simultaneous-presentation mode, typically shows a small transient over-representation of the unit effect b[U] relative to b[D] (Fig. 2b,c; Dotan & Dehaene, 2013).

4. Experiment 2: delaying decades or units

A possible concern about Experiment 1 is that it may not reflect the normal functioning of the cognitive system: the IDLE pattern may result from the experimental design, in which the unit digit was often delayed but the decade digit was never delayed. This design could have encouraged serial processing of the decade digit followed by the unit digit. Thus, the observed idle time window in the units processing pathway might be an artifact of Experiment 1 design. To control for this artifact, Experiment 2 delayed either the decade digit or the unit digit with equal probabilities. This symmetric design also allowed examining whether delaying the decade digit onset would delay its effect on finger movement, similarly to the corresponding effect we observed for units. Here we used a 0–100 number line, for which a binding error in any target number would yield a number in the valid range.

4.1. Method

The participants were 20 adults, aged 26;9 ± 4;5. The design was as in Experiment 1, but with a 0–100 number line and different onset times of the digits: each digit could appear either at $t = 0$ or at $t = 100$ ms (2 × 2 mixed design: a no-delay condition, in which both digits appeared at t = 0; delay decade; delay unit; and delay both digits). Each target number between 10 and 90 appeared twice per condition (648 trials). Like in Experiment 1, both X characters changed when the finger started moving (t = 0). In the no-delay condition, they changed immediately to the target (Fig. 4a). In the delay-decade and delay-unit conditions, at $t = 0$ only the decade or the unit digits appeared; the other digit changed at t = 0 into a lowercase ‘x' placeholder, and after 100 ms to the target. In the delay-both condition, the fixation changed to ‘xx' at t = 0, and after 100 ms to the target. The ‘0' placeholder was not used here because, unlike Experiment 1, it would have created an invalid two-digit number when the unit digit was displayed first.

The trajectory data was analyzed per condition using the two-stage regression analysis described in Section 2.4.1. To examine whether the decade quantification depends on the unit quantification and vice versa, the regression coefficients of each digit were compared between the four experimental conditions (no delay, decade delay, unit delay, both digits delayed). This comparison was done separately for the decade digit and the unit digit, using the two methods described in Section 2.4.2: comparing regression b values per time point, and calculating the delay between conditions.

4.2. Results

In the no-delay condition (Fig. 4b), the decade and unit effects were similar, with a small but significant over-weighting of the unit effect b[U] relative to the decade effect b[D] in a transient time window – again imputable to erroneous binding of digits to decimal roles, which increased the unit effect and decreased the decade effect. The different asymptotes of b[D] and b[U] (in $t \geq 700$ ms) show that at the end of the trial, the unit digit was slightly under-weighted relative to the decade digit.

The comparison between conditions is hereby described with respect to each digit.

4.2.1. Decade quantification

If the decade quantity is processed independently of the unit quantity, the decade effect on finger movement (b[D]) should depend only on the decade digit's onset time. This was indeed the case (Fig. 4c, Table 1). Delaying the decade digit onset by 100 ms caused a significant delay of about 100 ms in b[D] (no-delay vs. delay-decade and delay-unit vs. delay-both in Table 1). Conversely, delaying the unit digit onset by 100 ms only had a minor effect on the decade effect (no-delay vs. delay-unit and delay-decade vs. delay-both in Table 1). Thus, the decade digit was quantified and caused a corresponding finger movement as soon as it appeared, almost independently of the unit onset time.

4.2.2. Unit quantification

4.2.2.1. Delaying the unit digit results in an IDLE pattern. Experiment 1 showed an IDLE pattern in the unit effect: when delaying the unit digit onset, 35 ms of the delay were absorbed in a putative idle time window, and only additional delay was observed in finger movement. This pattern was replicated here: comparing the no-delay and delay-unit conditions (blue lines in Fig. 4d) showed that delaying the unit onset by 100 ms delayed its effect on finger movement b[U], and the size of the b[U] delay was 65 ms (Table 1), i.e., smaller than the visual stimulus delay. This is an exact replication of the two equivalent conditions in the D0 block of Experiment 1 (SOA = 0 and SOA = 100). Even the estimated idle time window duration was replicated (35 ms in both experiments).

4.2.2.2. Delaying the decade digit induces binding errors. Presenting the unit digit before the decade digit resulted in an unexpected pattern of results – extremely high b[U] values. Importantly, the b[U] peak value in the delay-decade condition (average b[U] over participants, peak = 1.37 at 550 ms) was significantly higher than in the no-delay condition (peak b = 0.93 at 700 ms; paired $t(19) = 2.35$, two-tailed $p = .03$, Cohen's d = 1.79). This difference in the peak b[U] is important because it cannot be explained by any delay model – i.e., the delay-decade condition did not *delay* the unit effect, but created an *exaggerated increase* in the unit effect. We explain this exaggerated unit effect b[U] as erroneous digit-to-role binding: presumably, the delay-decade condition induced many situations in which the unit was incorrectly bound to the decades role, and was consequently processed with 10 times its weight, thereby increasing its effect b[U]. A possible reason for the high rate of binding errors in the delay-decade condition is that in this condition there was a time window of 100 ms

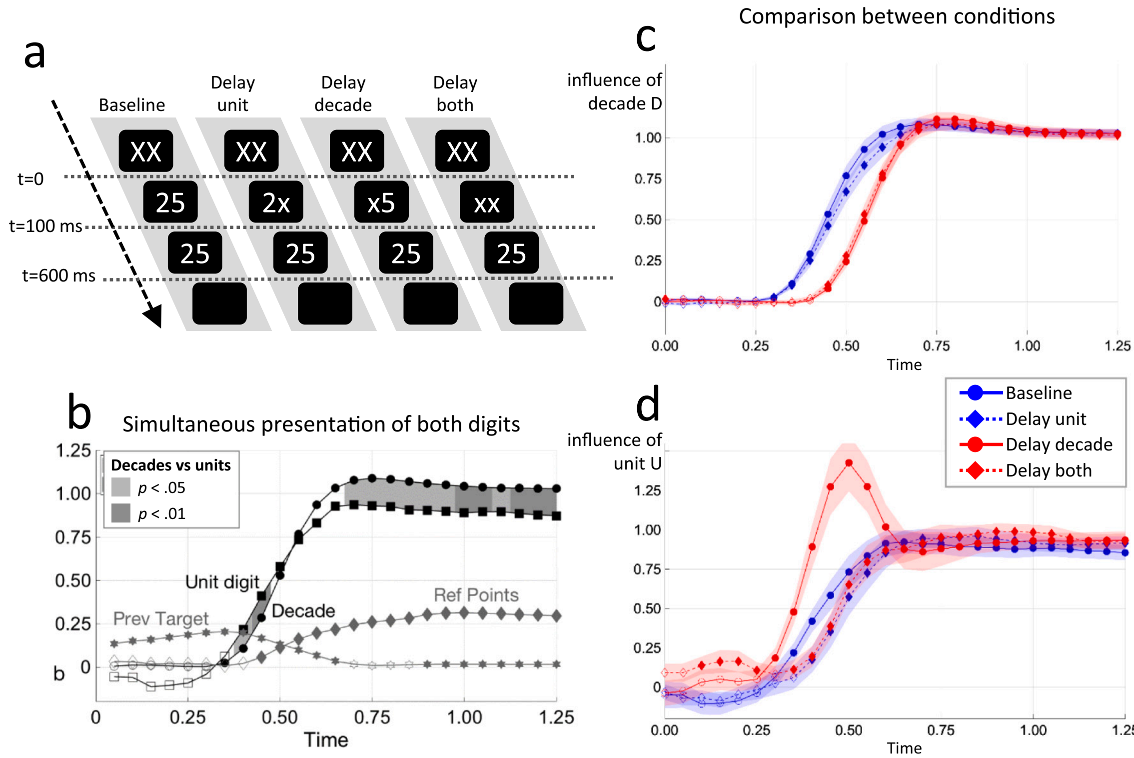

Fig. 4. Design and results of Experiment 2. (a) Task design: 4 mixed conditions. The fixation was 'XX'. At t = 0, each 'X' changed either immediately to the target digit, or to a lowercase 'x' and after 100 ms to the target digit. (b–d) Time course of the effects – same plot type as in Fig. 2. (b) When neither the decade digit nor the unit digit were delayed, their effects built up together and in almost 10:1 ratio, with a transient slight over-weighting of the unit digit. (c) The decade effect depended solely on the decade digit onset time, and was almost unaffected by delaying the unit digit. (d) Delaying the unit onset by 100 ms (no-delay vs. delay-unit) delayed its effect by only ~65 ms (IDLE pattern). Presenting the unit digit before the decade digit (delay-decade) resulted in an exaggerated unit effect.

Table 1
The regressions effects in Experiment 2 were compared between condition pairs using two methods: estimating the delay between two conditions, and examining whether the regression coefficients were significantly different between two conditions. The decade effect b[D] was coupled with the decade digit onset time, and was hardly affected by the unit onset. The unit effect b[U] showed the IDLE pattern (effect delayed by less than the lag in the digit onset).

Compared conditions [a]	Delay between conditions (ms) [b]	Significant differences between regression lines			
		When (ms)	t(19)	1-tail p	Cohen's d
b[D]					
None vs. decade	103	300–650	≥ 1.96	≤ 0.03	0.45–3.06
Unit vs. both	79	300–650	≥ 2.5	≤ 0.01	0.57–2.46
None vs. unit	18	400–650	≥ 2.1	< 0.03	0.47–1.48
Decade vs. both	−7				
None vs. both	98	300–650	> 1.9	≤ 0.03	0.44–2.94
b[U]					
None vs. unit	65	350–500	≥ 1.9	< 0.04	0.43–0.96
None vs. both	50	400–450	≥ 2.3	< 0.02	0.04–0.21

[a] Condition names: "none" = both digits appeared at t = 0; "decade", "unit", and "both" denote delay of the corresponding digit/s.
[b] The delays were computed based on the raw regression coefficients in the time window 350–650 ms. Similar results were obtained when using a wider time window, 250–750 ms.

during which only the unit digit appeared on screen, and may have been interpreted as decades.

To estimate the number of binding errors per condition, we focused on trials in which one digit was larger than 5 and the other digit was smaller than 5 (hereby, *critical trials*). The idea is that in these critical trials, a binding error would cause the finger to initially move to the incorrect side (rightwards for targets < 50 and leftwards for targets > 50), and this error may later be corrected. To detect this pattern in trajectories, we first dissected each trial into a series of *bends* – trajectory sections, at least 100 ms long, in which the finger continuously deviates either clockwise or counterclockwise. A binding error was defined as trial in which one of the first two bends started by pointing towards the incorrect side: implied endpoint > 60 for trials with target < 50, or implied endpoint < 40 for target > 50. We examined the proportion of trials with binding errors out of the critical trials. The binding error rate in the delay-decade condition (18.8%) was higher than in the other conditions (no delay: 5.6%, delay unit: 7.4%, delay both: 12.4%; for all 3 comparisons, paired $t(19) > 3.64$, two-tailed $p \leq .002$). Namely, the delay-decade condition induced more binding errors than the other conditions.

4.2.2.3. Delaying both digits. In the last condition we examined, both digits appeared simultaneously like in the baseline (no-delay) condition, but they did so only 100 ms after the finger started moving. Would the effect of the two digits also be delayed by 100 ms relative to the baseline condition? For the decade digit, this was indeed the case: the decade effect b[D] was delayed by almost 100 ms (Table 1). The unit effect b[U], however, was delayed by only 50 ms.

A possible explanation for this lower-than-expected b[U] delay is in terms of binding errors, which increase the unit effect b[U] and make it appear earlier than it really is (see Section 3.2.6). The degree of this bias in the estimation of b[U]'s timing is higher for conditions with more binding errors, so the lower-than-expected b[U] delay can be accounted for by a higher rate of binding errors in the delay-both condition than in the baseline condition. This was indeed the case (12.4%, in the delay-both condition versus 5.6% in the baseline condition, paired $t(19) = 4.29$, two-tailed $p < .001$; the binding error rate was computed as explained in Section 4.2.2.2). We conclude that the real delay in the unit effect b[U] was not 50 ms but was in fact larger – perhaps even 100 ms as expected.

4.3. Discussion of experiment 2

The main findings of Experiment 1 were replicated: first, when the two digits of a two-digit number were presented simultaneously, they influenced finger motion in a near-synchronous manner, with some overweighting of the unit digit relative to the expected 10:1 ratio. Second, the effect of the decade digit depended only on its presentation time, irrespectively of when the unit digit was displayed. Third, delaying the unit onset by 100 ms delayed its effect on finger movement by only 65 ms, presumably because the unit delay was partially absorbed in a 35 ms idle time window in the units processing pathway. Remarkably, both Experiment 1 and Experiment 2 yielded exactly the same estimate of an idle time window duration – 35 ms. Note that Experiment 2, in which the placeholder character was a letter ('x'), has replicated the Experiment 1 D0 block (with a digit placeholder) and not the D# block (with a symbol placeholder). Namely, the digit placeholder ('0') resembled the letter placeholder ('x'), not the symbol ('#'). In the General Discussion, we discuss the implications of this finding.

Presenting the unit digit before the decade digit resulted in a large increase in the unit effect on finger movement. This increase probably results from errors in binding the digits to decimal roles: when the unit digit appeared first, it was more often processed as if it was a decade digit and quantified 10 times its real value, causing the exaggerated unit effect on finger movement. Delaying both digits too increased the probability for binding errors, and correspondingly increased the unit effect on finger movement, even if to a lesser extent than in the delay-decade condition.

The decade-unit confusions may have been symmetric, i.e., it is possible that the decade digit was sometimes processed as a unit digit. Indeed, in Experiment 1 we observed errors in binding the decade digit via the analysis of the decade effect on finger movement. Here, conditions with binding errors did not show a reduced decade effect. A possible explanation is that binding errors are harder to detect via the decade effect than via the unit effect, because their effect on the decade digit is 10 times smaller than their effect on the unit digit.

5. Experiment 3: three-digit numbers

The IDLE pattern indicates the existence of a bottleneck process that causes an idle time window in the units processing pathway. What could this process be? The simplest explanation seems to be sequential processing of the digits: if the units can only be processed after the decades, this would create an idle time window in the units processing pathway. This explanation is in line with previous findings on sequential processing of digits in a multi-digit number: sequential effects were reported in number comparison and number reading (but only for numbers with 4 digits or more, Dotan et al., submitted for publication; Friedmann et al., 2010; Meyerhoff et al., 2012). However, as we shall see, Experiment 3 refutes the sequential-processing model.

The sequential-processing view has a major flaw: it predicts that when decades and units are presented simultaneously, the decade effect on finger movement should precede the unit effect. This was not the case: in simultaneous presentation, the decade and unit effects consistently build up in parallel (Fig. 2b,c; Fig. 4b; Dotan & Dehaene, 2013, 2016). To explain this parallel pattern, the sequential-processing view could assume the existence of another factor that masks the decade-unit delay – e.g., binding errors (as we saw in Figs. 2b, c, and 4b, binding errors seem to exist even in simultaneous-presentation condition). Binding errors increase the unit effect and decrease the decade effect. Because larger effect and earlier effect are almost indistinguishable in the analysis of a given trajectory, the increased unit effect may seem like an earlier effect, and the decreased decade effect may seem like a delayed effect – i.e., binding errors create an artificial apparent negative delay between the decade and unit effects. This negative delay could, by pure coincidence, just happen to be of the same size as the decade-unit delay, in which case the effect of binding errors would exactly cancel out the delay caused by sequential processing.

Such a coincidence seems unlikely, especially given that the nearly-simultaneous buildup of the decade and unit effects was replicated in 12 experiments (Experiment 1, Experiments 2, Dotan & Dehaene, 2013, and 9 experimental conditions in Dotan & Dehaene, 2016). Nevertheless, to examine this further, we ran the number-to-position mapping task with 3-digit numbers, with simultaneous presentation of all digits. If we observe parallel effects of all digits (units, decades, and hundreds) in this experiment too, the serial-processing view would have to assume that the delays between digits are coincidentally exactly canceled out by a complex pattern of binding errors across units, decades, and hundreds. Such a coincidence over 3 digits seems virtually impossible. Thus, the sequential-processing view would not be able to account for overlapping regression effects of the 3 digits in this experiment.

5.1. Method

The participants were 20 right-handed adults, aged 25;10 ± 4;1. The task design was like in the previous experiments, yet without delays – all digits always appeared simultaneously at $t = 0$. The number line ranged from 0 to 400, and each number between 0 and 400 was presented once.

We applied the regression analysis method as above, with 5 predictors: the unit digit U, the decades D (0, 10, 20, …), the hundreds H (0, 100, 200, 300 or 400), the previous target N-1, and the spatial-reference-point-based bias function SRP (Section 2.4.1), modified to match the 0–400 number line length.

5.2. Results and discussion

The effects of the two leftmost digits (hundred and decade) built up in almost exact parallel in the expected 1:10 ratio, with a transient small overweighting the decade digit (Fig. 5). These results replicate the parallel buildup of decades and units observed in the previous experiments. The unit effect was higher than the expected 1:10:100 ratio versus the decade and hundred effects during a transient time window; this replicates the previous findings that the unit effect b[U] was transiently higher than the decade effect b[D] when the digits were presented simultaneously (Experiments 1 and 2, Dotan & Dehaene, 2013). The elevated b[U] values could be explained by binding errors, which increased the unit effect b[U] relative to the decade (b[D]) and hundred (b[H]) effects. Binding errors can easily explain why the largest bias was observed for the units effect (and not for the decades or hundreds effects), and why the b[U] bias was larger here than in experiments with two-digit numbers: in 3-digit numbers, each error of binding the unit to the hundreds location would result in processing the unit digit with 100 times its real weight, so even a small rate of units-hundreds binding errors would considerably inflate b[U].

The almost perfect alignment of the hundred and decade effects is

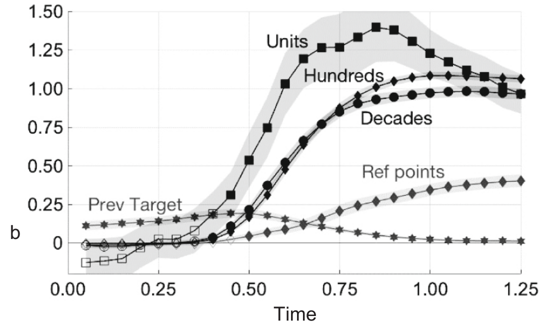

Fig. 5. Regression results of Experiment 3, in which 3-digit numbers were presented with all digits simultaneously (same plot type as in Fig. 2). The hundred and decade effects build up in parallel over time, replicating the findings of two-digit numbers. The unit effect is massively over-weighted, compatible with the presence of occasional errors of binding the unit digit either to decades or to hundreds.

hard to reconcile with the sequential-processing model. For two-digit-number experiments, the sequential processing model had to assume that the apparent delay caused by binding errors (due to increased unit effect and decreased decade effect) was exactly identical with the real decade-unit delay imposed by sequential processing. For the present results, the sequential model must also assume that the apparent delay caused by binding errors to the increase in the decades effect relative to the hundreds effect was exactly identical with the real hundreds-decades delay imposed by sequential processing. This seems hardly plausible. The results therefore reject the notion of purely sequential processing of the digits. Rather, the quantities provided by the different digits appear to feed finger movement in parallel, yet – as shown by the previous experiments – with a small idle time window during which the units can be delayed without any consequence on behavior. In the General Discussion, we present a model that formalizes those hypotheses and can account for the results.

6. General discussion

6.1. The properties of number-to-quantity conversion processes

This study examined how the decade and unit digits of a two-digit number interact when transforming the number to quantity. We used a number-to-position mapping task, which forces the participants to convert the digit string to quantity, and we monitored the finger trajectories to get an insight into the temporal dynamics of this quantification process.

The main findings with respect to two-digit numbers, which any theory of number comprehension should account for, can be summarized as follows. First, *when decades and units were simultaneously presented, their effects on finger movement built up in parallel*, with a small over-weighting of the unit digit relative to the expected decade-unit ratio of 10:1. Second, *the timing of the decade effect depended only on the decade digit*: delaying the presentation of the decade digit by Δt induced an identical delay Δt in the effect of the decade quantity on finger movement, whereas delaying the presentation of the unit digit hardly modified the decade effect on finger movement. Third, unlike the decade effect, *the timing of the unit effect depended on both digits*. Delaying the unit digit by up to about 35 ms had almost no effect on finger movement (i.e., the unit effect did not solely depend on the unit onset time). Delaying the unit digit by Δt > 35 ms induced a delay in the effect of the unit quantity on finger movement, and the size of this delay was smaller than Δt by about 35 ms. Last, *the relative weighting of decades and units was sometimes biased*. Imposing a decade-unit onset discrepancy resulted in an amplification of the amplitude of the unit digit and, correspondingly, a reduction in the amplitude of the decade digit. These weighting errors were largest in Experiment 2 and in the D# block in Experiment 1 – perhaps because in both cases, the two-digit stimulus was preceded by a transient single-digit number.

In turn, these findings afford several conclusions:

1. **Immediate processing of the decade digit.** The finding that the decade effect depended solely on the decade digit onset time indicates that the decade digit is processed as soon as it appears, without having to wait for the unit digit. This means that the quantity system, and the decision stage following it, operate as a continuous integrator of the information carried by the various digits. In particular, when the decade and unit information become available at different times, the finger aiming can be initially guided by just the decade digit and be corrected later, when the units information arrives. The trajectory-tracking paradigm is therefore sensitive enough to track the time course of the processing of the two digits with a high temporal resolution – and this time course is incompatible with any model assuming that motor gestures are only programmed once complete information has been obtained. The continuous nature of movement programming, and the ability to capture it with finger tracking, is supported by several previous studies, both with continuous spatial response like here (Dotan, submitted for publication; Pinheiro-Chagas et al., 2017) and with binary responses (Dotan et al., 2018; Erb et al., 2016; Finkbeiner et al., 2008; Finkbeiner et al., 2014; Finkbeiner & Friedman, 2011; Freeman et al., 2011; Marghetis et al., 2014; Santens et al., 2011; Song & Nakayama, 2009, 2008a, 2008b), and was also modeled mathematically (Alonso-Diaz et al., 2018; Friedman et al., 2013; also see a review of the continuous relation between the decision processes and manual movement in Gallivan et al., 2018 and in Dotan et al., 2019).
   Our findings clearly refute any model that assumes a single decision point for number comprehension and/or movement – in particular the lexical model presented in the Introduction, which postulates that the entire two-digit string is recognized and mapped to a whole-number quantity, and the max model, which postulates that the finger deviates only after both digits have been quantified and merged. Such models cannot account for the finding that in some conditions, the finger first moved according to the decade quantity whereas the unit effect kicked in later. This discrepancy between the decade and unit effects implies either a continuous updating or, at least, two movement decisions, one based on the decade and another based on the full two-digit number.
2. **Idle time window in the units processing pathway.**The unit effect showed a completely different pattern: any lag of Δt in the unit digit onset time resulted in a delay of Δt - 35 ms in the unit effect, and a lag smaller than 35 ms caused no delay. This indicates that the pathway for processing the unit digit contains an idle time window of approximately 35 ms, during which the unit digit appears to be waiting for the end of a bottleneck process initiated by the decade digit. Below, we propose a model that specifies what this process might be.

The pattern of the unit effect clearly shows that the digits are not processed fully independently and in parallel, as proposed by the parallel-decomposed model presented in the Introduction. If this were the case, the unit digit should have been processed immediately as it appeared, with no idle time window, and any delay in the unit onset would be fully reflected in its effect on finger movement.

The asymmetry between decades and units cannot be attributed to the fact that the decade digit was more informative than the unit digit in the specific case of our number-to-position mapping task. For any task with two-digit number stimuli, in which the response is based on the whole-number quantity, the properties of the decimal system dictate that the effect of the decade digit should be 10 times larger than the effect of the unit digit, and this asymmetry may bias the behavior in favor of the decade digit. In such tasks – our number-to-position task and several other tasks – the decade digit would always be more informative about the correct response than the unit digit.

3. **Errors in binding digits to decimal roles.** In Experiments 1 and 2, all stimuli were two-digit numbers. When the transient stimulus was a single-digit number, we observed a bias in the weights of the digits – decades were underweighted and units were overweighed – and this bias increased as a function of the duration of displaying the single-digit transient stimulus. Our interpretation of this pattern is that the decade and unit digits are sometimes bound to incorrect decimal roles. Thus, the decade digit is sometimes processed as units, with 1/10 its weight, and correspondingly the unit digit is sometimes processed as decades, with 10 times its weight. These binding errors are facilitated by displaying a digit on screen alone rather than as part of a two-digit number.

Note that binding errors do not necessarily have to be full-fledged errors: our data is also consistent with the possibility of "binding ambiguity", as a result of a transient positional uncertainty in the location of each digit (such uncertainty was also described with respect coding the positions of letters in words, Davis, 2010; Friedmann & Gvion, 2001; Friedmann & Rahamim, 2007). One possible mechanism that can account for binding ambiguity is based on the assumption that digit-to-role binding is implemented by activating one of two separate representations of the digit – one reflecting its value as units, another reflecting its value as decades (Huber et al., 2016). Within this model, binding ambiguity occurs when a digit activates, to some extent, both the unit value and the decade value.

The notion of binding errors may be relevant to the so-called compatibility effect: when participants are asked to judge which of two 2-digit numbers is numerically larger, their response times are affected by the overall numerical distance between the two numbers, which is known to affect reaction times (Moyer & Landauer, 1967), but also by the numerical distance between the two unit digits, which is irrelevant. The specific finding is that comparing pairs in which the unit comparison is compatible with the whole-number comparison (e.g., 34 vs. 59) is faster than comparing incompatible pairs (39 vs. 64), even when controlling for the overall numerical distance between the compared numbers (Nuerk et al., 2001). The common explanation of this effect is that participants do not compare the two whole-number quantities, but rather they separately compare the two decade digits and the two unit digits, and the unit comparison affects the reaction time although it is irrelevant. Binding errors offer an alternative account of the compatibility effect: on incompatible trials only, transient erroneous binding of the unit digit to the decade role would result in an incorrect result of the whole-number comparison, thereby slowing the responses in incompatible trials. Consistent with this idea, presenting the unit digit before the decade digit increases both binding errors (as observed here) and the compatibility effect (Knops, 2006). Future studies may further probe the putative relationship between binding errors and the compatibility effect.

4. **Parallel decade and unit processing in simultaneous presentation.** When the digits were presented simultaneously, they contributed to the quantity in parallel and in 10:1 ratio. This finding is extremely robust – it was observed in several experiments with two-digit numbers (here and in Dotan & Dehaene, 2013, 2016), and even between the hundred and decade digits of three-digit numbers (Experiment 3). In these experiments, the unit digit was often slightly overweighed relative to the other digits, suggesting perhaps that some degree of binding errors existed even when the digits were displayed simultaneously.

   Even given these findings of parallel decade-unit effects, the existence of binding errors could have conceivably supported the possibility that decades and units are actually processed serially. Serial processing should result in serial effects on finger movement – first the decade effect, then the unit effect – but this serial pattern could be masked by binding errors: such errors increase the unit effect relative to the decade effect, making the unit effect appear earlier (because in our regression analyses, a larger effect is almost indistinguishable from an earlier effect). However, our data speaks against this serial-processing model. Complete masking of the serial pattern would require that the apparent delay induced by binding errors, would have exactly the same duration as the real delay, induced by serial processing. To accommodate the robust finding of parallel effects of the different digits, a serial-processing model would have to explain why the real and virtual delays consistently have identical durations, in two-digit and three-digit numbers. At present, we see no such explanation.

   This remarkable degree of coordination between decades and units is a robust finding in expert readers, yet apparently it is not a trivial ability for the quantification system. Children, even not very young ones, show a completely different pattern. In a previous study, we examined a group of 4th grade children in the number-to-position task with simultaneous presentation of the two digits (Dotan & Dehaene, 2016, Experiment 4). We observed a discrepancy of 50–100 ms between the decade and unit effects on finger movement, suggesting that the children were processing the digits serially, first the decades, then the units. Thus, even after learning to read multi-digit numbers, the cognitive system may require several more years to fully automatize the parallel processing of digits. Indeed, word reading follows a similar developmental pattern, with an initial serial letter-by-letter stage followed, after 2–3 years, by automatic parallel processing of up to ~8 letters (Cohen et al., 2008; New et al., 2006; Zoccolotti et al., 2005). Furthermore, the adult ability for parallel processing of all digits can be impaired following brain damage, making an adult exhibit a serial pattern (Dotan et al., 2014).

   Our findings clearly indicate parallel processing of 2-digit numbers, and even for 3-digit numbers (at least for the hundred and decade digits; with respect to the unit digit, our paradigm is not accurate enough to arbitrate whether the processing was completely parallel or with a small degree of seriality). In contrast, in longer numbers – with 4 digits or more – at least some of the processing stages are serial (Dotan et al., in preparation; Friedmann et al., 2010; Meyerhoff et al., 2012). Some evidence for serial processing was shown even for 3-digit numbers (Bahnmueller et al., 2016), but only for some individuals, in a particular task (which may facilitate serial processing more than the task used here), and in German (which may facilitate serial processing due to the characteristics of verbal numbers in this language).

Methodologically, the present data joins several studies in showing the ability of finger tracking to dissect the temporal aspects of cognitive processes (Dotan et al., 2019), in particular when using manipulations that change the stimulus during a trial (Dotan et al., 2018). Finger tracking therefore has the potential of becoming a useful and simple behavioral tool to dissect cognitive processes temporally, and may even provide sufficient accuracy to serve as a diagnostic tool for single individuals (Dotan et al., 2014). At the same time, the present study also demonstrates a limitation of this paradigm: it confounds time and space, such that an earlier effect on the finger movement is almost indistinguishable from a larger effect on the finger movement. In our task, this limitation confounded the effects of delaying a digit and of binding errors. In the future, newer analysis methods may perhaps be able to address this limitation (e.g., single-trial analyses, Dotan et al., 2018).

6.2. A revised model of number-to-quantity conversion

Our findings refute all the number-to-quantity conversion models presented in the Introduction. The lexical model and the max model predicted that the decade and unit digits would have parallel effects even if one digit appears before the other, and this prediction was

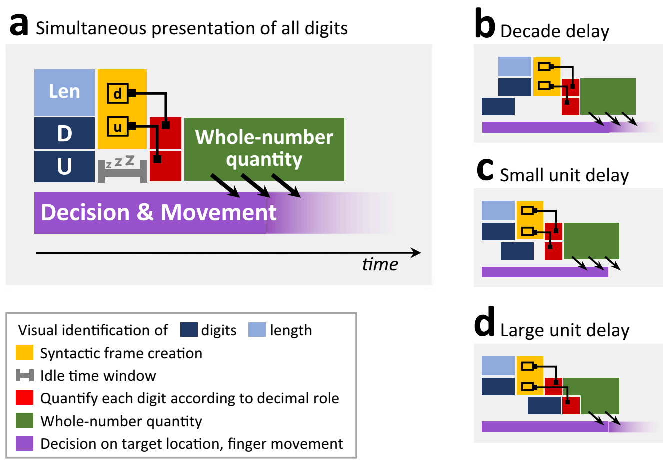

Fig. 6. Proposed model of two-digit number comprehension. The appearance of the leftmost digit triggers the identification of number length (light blue) and the subsequent formation of a *syntactic frame* (yellow). In parallel, the two digits are visually identified (dark blue). The formation of the syntactic frame imposes a bottleneck: while the frame is being created, the processing of digits is idle (grey). This idle time window lasts about 35 ms. Once the frame is ready, each digit can be assigned to a decimal role, quantified accordingly (red), and merged into a whole-number quantity representation (green), which continuously feeds the decision process that drives the finger movement (purple). Each panel illustrates the delays predicted in each stimulus condition: (a) When the digits are presented simultaneously, they simultaneously integrate into the whole-number representation and affect the finger movement. (b) Delaying the decade digit onset by Δt delays the syntactic frame initiation, and consequently should delay the quantification of both digits by the same Δt. In our experiments, the unit delay in this case was masked by a large overarching effect of unit amplification, which we attribute to an erroneous assignment of digits to their decimal roles. (c) Short delays (Δt < 35 ms) in the onset of the unit digit of a two-digit number do not affect the syntactic frame creation, so they are fully absorbed by the idle time window and have no effect on finger movement. (d) Longer delays in the unit onset are partially absorbed by the idle time window, so the unit effect is delayed, but by less than Δt. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

refuted. The parallel-decomposed model predicted that the effect of each digit on finger movement would depend only on that digit's appearance time, and not on the appearance time of the other digit, and this prediction too was refuted for units digits.

To account for the data, we therefore propose a novel detailed model of multi-digit number comprehension (Fig. 6). We are not claiming that this is the only possible model for the data, but rather, that it can account for all of the complex data patterns that we observed, and is also natural from a rational analysis of the problem faced by the brain. The new model is an enhancement of the model proposed in Dotan and Dehaene (2016), which referred to several stages in the number-to-position task (symbol identification of the digits, quantification, decision, and pointing), and where we proposed a detailed Bayesian analysis of the decision stage. The present data provide a more refined picture of the first two stages, identification and quantification, during which subjects must convert the digit shapes on screen into a representation of the corresponding quantity.

A two-digit number must be initially processed as two independent shapes, because the two digits are projected to distinct retinotopic locations. At some point, however, the system must stop processing the digits independently and bind them to distinct roles, one being the leftmost decade digit (worth 10) and the other being the rightmost unit digit (worth only 1). This process is not trivial because the absolute location of the digits may vary relative to the fixation point, and indeed the order of digits is encoded by a dedicated process (Dotan & Friedmann, 2018; Friedmann et al., 2010; for an analogous distinction in word reading, between spatial location of letters and their within-word positions, see Dehaene et al., 2004). The digit identities, their relative order, and the number of digits provide sufficient information to assign the digits to their decimal roles as decades or units. We propose that this is done by binding the digits to a *syntactic frame* of the multi-digit number. The term "syntactic frame" is a concept borrowed from models of digit-to-verbal number transcoding, where it was found indispensable to account for the errors made by various patients with brain lesions (Cohen & Dehaene, 1991; McCloskey, 1992; McCloskey et al., 1986). Here, it refers to a mental representation of the structure of the multi-digit number that takes into account its overall length, but not yet the specific digit values. The syntactic frame of a single-digit number has a single placeholder 'u' for the unit quantity; the syntactic frame of two-digit numbers has two placeholders – 'd' for the decade quantity and 'u' for the unit quantity; and so on. Once the syntactic frame is created (Fig. 6a, yellow), each digit is quantified according to its decimal role (red): the value of the 'd' digit is interpreted as a decade, i.e., multiplied by 10 compared to the value of the 'u' digit. The binding of each digit to its decimal role may occur as part of the syntactic frame creation or after it. An interesting possibility is that that digit-to-role binding and the syntactic frame are one and the same: the "syntactic frame" could be simply the binding of each visual input channel, containing the visual information of a single digit in a particular retinal location, to a "digit detector" that identifies the digit (Davis, 2010; Dehaene et al., 2005; Whitney, 2001), and then to an output channel that sends the identified digit to quantification according to a particular decimal role. After computing the quantity for each digit, the per-digit quantities are combined into a whole-number quantity (green) and fed to the decision process that determines the target location and eventually drives the finger movement (purple).

The selection of an appropriate syntactic frame requires knowing how many digits the number has. The model postulates that this information is extracted by a dedicated number-length encoder process (Fig. 6a, light blue color), which is a part of the visual analyzer of digit strings. Presumably, the visual analyzer serves any task that requires parsing digit strings. When reading numbers aloud, the number-length detector is one of several processes that encode the number's structure, and this may allow the verbal system to prepare itself for saying a verbal number with this structure (Cohen & Dehaene, 1991; Dotan & Friedmann, 2018). The same idea may apply here: a dedicated process that extracts the number's structure (its length) may be needed to initiate the appropriate quantity syntactic frame as quickly as possible, so not to delay the subsequent processing stages. In word reading too, Davis (2010) has proposed an analogous process that detects the number of letters in a word, and initializes the letter-identification processes once the leftmost letter was detected. Still, note that the mechanics underlying this initialization process, and even its goal, may be different for numbers and words, which have two separate visual analysis systems (Abboud et al., 2015; Dotan & Friedmann, 2019).

Our model assumes that decades and units can be processed asynchronously and independently at all stages, except a single bottleneck point – the creation of the syntactic frame. Crucially, the identification of number length, and consequently the initiation of the syntactic frame, is triggered by the decade digit in the two-digit number (or, more generally, the leftmost digit in numbers of arbitrary length). This explains why any delay in the decade digit impacts on the syntactic frame initiation and consequently on the binding and quantification of both digits (Fig. 6b). Assuming that the syntactic frame takes about 35 ms to form, the model can also account for the IDLE pattern in two-digit numbers: a short delay (< 35 ms) in the visual presentation of the unit digit has no effect (Fig. 6c) because this delay is entirely absorbed by the contemporaneous formation of the syntactic frame. As a result, the timing of the unit quantification process remains unchanged. Only a delay of Δt > 35 ms in the unit digit onset is large enough to make the visual identification process end after the syntactic frame initiation, thereby delaying the unit quantification by Δt – 35 ms (Fig. 6d).

6.3. Additional aspects of the proposed model

6.3.1. The number-length detector

The number length detector aims to encode only the number length, and as such it does not rely on encoding the precise identity of either of the digits. The number-length detector does require, however, that *some* digit would be present at the decade position. It may also rely, to some extent, on the presence of some digit in the unit position, so the process may occasionally break down if the unit position is occupied by a non-digit placeholder. This may explain why the IDLE pattern was not observed in the D# condition in Experiment 1.

Interestingly, the IDLE pattern was observed in Experiment 1 when the placeholder character was '0' but not when it was '#', and was observed also in Experiment 2 when the placeholder character was not a digit but the letter 'x'. This may indicate that the number-length detector is sensitive only partially to the type of the character shown: it does not distinguish between letters and digits, but it does distinguish between letters/digits and symbols. This insensitivity to the character being a letter or a digit agrees with studies showing that the early processing stages of visual analysis, before identifying the precise identify of each character, do not distinguish between letters and digits (McCloskey & Schubert, 2014). The number-length detector, which is hypothesized to be an early-stage process, presumably relies on the information arriving from these early processing stages, and therefore it too may not distinguish between letters and digits. In contrast, even the early-stage visual processes, and consequently the number-length detector, may still distinguish between letters and digits on the one hand and other symbols on the other hand. This may explain why the experiments with letter and digit placeholders yielded an IDLE pattern, whereas the symbol placeholder did not. This "hierarchy" of character types, in which visual processes treat letters and digits in similar manners but symbols in a different manner, was reported by several previous studies of visual character processing (Chanceaux & Grainger, 2012; Grainger et al., 2010; Tydgat & Grainger, 2009).

6.3.2. Similarity to other models

The model proposed here shares several aspects with other models of number processing. The model distinguishes between processes that identify the visually-presented digits and processes that create the quantity representation (see a review in Dehaene et al., 2003). Within the visual identification mechanisms, the model assumes a dedicated process that identifies the number length – a process that was also proposed in models of number reading (Cohen & Dehaene, 1991; Dotan & Friedmann, 2018). Furthermore, the binding of each digit to a decimal role is analogous to the mechanism introduced by McCloskey and colleagues, where multi-digit numbers are represented by associating a sequence of 0–9 values with corresponding powers of 10 (McCloskey, 1992; McCloskey et al., 1986).

6.3.3. Holistic versus decomposed representation of quantities

Our model offers to settle a long-lasting debate about whether the quantity representation of two-digit numbers is holistic or decomposed. Some studies argued that a holistic quantity of the two-digit number is created (Dehaene et al., 1990; Dotan & Dehaene, 2013; Fitousi & Algom, 2006; Reynvoet & Brysbaert, 1999), whereas others argued for decomposed single-digit quantities (Meyerhoff et al., 2012; Moeller et al., 2009; Nuerk & Willmes, 2005). Our model reconciles these views by assuming that whenever we convert a two-digit number into quantity, each digit is first quantified independently, but the two per-digit quantities are quickly merged to a whole-number quantity representation (with occasional binding errors).

Different tasks and experimental designs may encourage participants to rely on either of these two representations – the per-digit quantities or the whole-number quantity. For example, if the integration of per-digit quantities into a whole-number quantity can only be done for one multi-digit number at a time, a task that presents several multi-digit numbers simultaneously may encourage the participants to avoid the conversion into whole-number quantity and instead rely on the per-digit quantities (Dehaene et al., 1990; Nuerk & Willmes, 2005; Zhou et al., 2008). Future studies may elaborate further on the transition between these two representations and on the limitations of each.

6.4. Conclusion

Our data suggests that converting 2-digit and 3-digit numbers to quantity involves a mixture of serial and parallel processing. On one hand, there is a bottleneck process that imposes serial processing. On the other hand, all other stages appear to process the digits in a parallel and asynchronous manner, both before and after the bottleneck – i.e., not only in the visual recognition of digits, but even in the deeper processing stages of conversion to quantity. This remarkable degree of parallel processing is not trivial, and indeed it apparently takes years to develop (Dotan & Dehaene, 2016). Future studies may investigate its stages of development, as well as whether certain kinds of input are needed to facilitate this development.

The architecture we propose, a syntactic bottleneck process (the creation of a syntactic frame) surrounded by several parallel processes of visual recognition and quantification, suggests that the quantification process is not driven by single-digit processing but by the syntactic processes that handle the number's decimal structure. This central role of structural processing is not unique to number comprehension: in number reading too, several processes explicitly represent the number's decimal and verbal structure, and these structural mechanisms apparently drive the reading process (Dotan & Friedmann, 2018). Structural processing prevails also in word reading: the morphological structure of words is explicitly represented in several processing stages, from visual analysis processes to phonological production processes (Beyersmann et al., 2011; Dotan & Friedmann, 2015; Kohn & Melvold, 2000; Rastle et al., 2004; Reznick & Friedmann, 2015). It may be the case that when faced with the need to convert compound stimuli from one representation to another, the cognitive system consistently tends to mediate the conversion process via deep representations of the compound stimulus structure.

CRediT authorship contribution statement

Dror Dotan: Conceptualization, Methodology, Software, Formal analysis, Investigation, Writing - original draft, Writing - review & editing, Project administration.**Stanislas Dehaene:** Conceptualization, Methodology, Writing - original draft, Writing - review & editing, Formal analysis, Supervision, Funding acquisition.

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Declaration of competing interest

The authors have declared that no competing interests exist.

Appendix A. General patterns of performance

The analyses in this study focused on the dynamics of finger movement within a trial. As background, we also analyzed several summary measures per trial, which are detailed in Table A.1. The *endpoint bias* is the difference between the endpoint and the target number (positive value = rightward bias). *Endpoint error* is the absolute value of a specific trial's endpoint bias. *Movement time* is the duration from the time when the finger started moving (which is also when the stimulus appeared in undelayed conditions) until it reached the number line. In all experiments, these measured showed similar values to previous experiments in this paradigm (Dotan & Dehaene, 2013, 2016). The only major difference was the slower movement time in Experiment 3, indicating unsurprisingly that 3-digit numbers are processed more slowly than 2-digit numbers.

Table A.1
End-of-trial-time measures. Numbers in parentheses indicate the standard deviation of the per-participant means.

	Exp. 1, D#	Exp. 1, D0	Exp. 2	Exp. 3
% of failed trials	5.4 (4.5)	4.6 (3.5)	2.8 (2.6)	4.3 (3.5)
Movement time (ms)	1012 (161)	980 (137)	1036 (177)	1272 (186)
Endpoint bias [a]	−0.73 (0.88)	−0.75 (0.8)	−1.36 (0.87)	−5.81 (4.71)
Endpoint error [a]	2.89 (0.84)	2.83 (0.81)	4.68 (1.17)	23.8 (7.58)

[a] Endpoint bias and error use each Experiment's number line scale (0–60, 0–100, or 0–400).

We also examined the overall patterns in the regression curves. The decade and unit regression effects were analyzed in detail in the main text. Table A.2 provides basic information about the two other regression effects – the previous target (N-1) and the spatial-reference-points-based bias function (SRP). In all experiments, the regressions showed the patterns typical to our number-to-position paradigm (Dotan & Dehaene, 2013, 2016):

- A significant effect of the previous trial in the early trajectory parts (until 600–700 ms, with the peak effect around 350–400 ms)
- A spatial-reference-points bias (SRP) effect in the late trajectory parts (starting from 450 to 600 ms).
- A dominant effect of the target number (both digits).

Table A.2 also shows that in the trajectory endpoints, the relative weights of the decade and unit digits (or, in Experiment 3, of the hundred and decade digits) were close to a 10:1 ratio.

Table A.2
General patterns in the regressions.

	Exp. 1, D#	Exp. 1, D0	Exp. 2	Exp. 3
b[N-1]				
Peak b value	0.20–0.22	0.23–0.26	0.20–0.21	0.19
Peak time point	350–400	350–400	350–400	500
b > 0.05 until (ms)	600–700	650–750	600–700	850
b[SRP]				
Significant from (ms)	450–600	450–550	450–500	500
Endpoint b value	0.29–0.32	0.27–0.31	0.29–0.30	0.39
b[U] / b[D] ratio at endpoints	1.01–1.06	0.87–1.03	0.85–0.91	0.92 [a]

[a] The ratio shown for Experiment 3 is b[D]/b[H].

Appendix B. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.cognition.2020.104387.

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