List of mathematical and non-mathematical statements

1 Analysis

1.1 True :

Statement 1. The Fourier series expansion of a continuous and piecewise $\mathcal{C}^1$ function f converges pointwise to f.

Statement 2. Any locally polynomial function from $\mathbb{R}$ to $\mathbb{R}$ is polynomial.

Statement 3. The function $\frac{1}{\Gamma(z)}$ admits an analytic continuation to the whole complex plane.

Statement 4. Any compact topological group admits a unique probability measure invariant under left-translations.

Statement 5. The set of test functions is dense in every space L^p, for $p \geqslant 1$.

Statement 6. A smooth function whose derivatives are all non-negative is analytic.

1.2 False :

Statement 7. The spaces L^p are separable.

Statement 8. The Fourier transform is an isometry from $L^1(\mathbb{R}^n)$ onto itself.

Statement 9. The topological dual of $L^\infty(\mathbb{R})$ is $L^1(\mathbb{R})$.

Statement 10. An inequality between two functions remains valid for their primitives.

Statement 11. There exists a continuous map from the unit ball into itself without any fixed point.

Statement 12. The distributional derivative of the Heaviside step function is the Heaviside step function.

1.3 Meaningless :

Statement 13. Any Dirac Heaviside function admits a Taylor expansion in L^p.

Statement 14. The space $L^1(\mathbb{R}^n)$ admits a locally polynomial, separable and analytic measure.

Statement 15. In finite measure, the series expansion of the roots of a holomorphic map is reflexive.

Statement 16. The topological dual of a Fourier series admits an analytic continuation.

Statement 17. The trace of the unit ball diverges for some $p \notin \{1, \infty\}$.

Statement 18. Any compact polynomial space is isometric to a unique space L^p.

2 Algebra

2.1 True :

Statement 19. A square matrix with coefficients in a principal ideal domain is invertible if and only if its determinant is invertible.

Statement 20. For even n, any sub-algebra of $M_n(\mathbb{C})$ of dimension ≤ 4 admits a non-trivial centralizer.

Statement 21. The square matrices with coefficients in a field that are equivalent to a nilpotent matrix are the non-invertible matrices.

Statement 22. Up to conjugacy, there only exists 5 crystallographic groups of the plane.

Statement 23. There exists a 13-dimensional algebra of 4×4-complex matrices.

Statement 24. $\mathbb{Q}$ can be canonically embedded into any field of characteristic zero.

2.2 False :

Statement 25. There exists a group of order 169 whose center is reduced to one element.

Statement 26. Any matrix with coefficients in a principal ideal is equivalent to a companion matrix.

Statement 27. A group of which all proper subgroups are abelian is abelian.

Statement 28. In the algebra $M_n(\mathbb{C})$, if two sub-algebras commute, the sum of their dimensions is not greater than n^2.

Statement 29. Any square matrix is equivalent to a permutation matrix.

Statement 30. There exists an infinite order group that admits a finite number of sub-groups.

2.3 Meaningless :

Statement 31. Any square invertible ring admits a hexadecimal expansion.

Statement 32. Any matrix with cardinality greater than 3 is factorial.

Statement 33. The field of fractions of an immatricial algebra is embedded in the space of projections.

Statement 34. Any algebra of dimension not greater than 4 is a linear combination of three projections.

Statement 35. There only exists 5 nilpotent canonically additive groups.

Statement 36. The field $\mathbb{R}[i]$ admits a free noetherian centralizer over $\mathbb{Q}$.

3 Topology

3.1 True :

Statement 37. A finite left-invariant measure over a compact group is bi-invariant.

Statement 38. The boundary of the Cantor set equals itself.

Statement 39. There exists non-discrete spaces whose connected components are reduced to one point.

Statement 40. The union of a family of pairwise non-disjoint connected subsets of $\mathbb{C}$ is connected.

Statement 41. Any locally finite bounded set of $\mathbb{R}$ is finite.

Statement 42. The quotient of a topological group by its identity component is totally disconnected.

3.2 False :

Statement 43. Any continuous bijection between two Hausdorff spaces is a homeomorphism.

Statement 44. There exists a continuous function from the unit sphere onto itself without any fixed point.

Statement 45. Any convex compact set of a euclidean space is the intersection of a family of closed balls.

Statement 46. In any topological space, every subspace homeomorphic to an open set is also an open set.

Statement 47. Every complete graph can be embedded into the unit sphere of $\mathbb{R}^3$.

Statement 48. Any inifinite set of real numbers admits at least one accumulation point.

3.3 Meaningless :

Statement 49. Every non-decreasing morphism of the Cantor set is conjugated to a homeomorphism of the unit ball.

Statement 50. Every finite measure on a Hopf algebra is locally modelled on the Haar measure.

Statement 51. The boundary of a homeomorphism has empty interior.

Statement 52. A subset of $\mathbb{C}$ is always left-invariant and right-continuous.

Statement 53. The graph of the completion of a compact group is dense in a partially connected open set.

Statement 54. Evey non-countable measure is the intersection of a family of compact groups.

4　Geometry

4.1　True :

Statement 55. Any vector field on an even-dimensional sphere vanishes.

Statement 56. The eccentricity of a rectangular hyperbola equals $\sqrt{2}$.

Statement 57. In an ellipse, the ratio of the distance from the center to the directrix equals half the major axe over the eccentricity.

Statement 58. The set of points that are equidistant from two given disjoint lines of $\mathbb{R}^3$ is an hyperbolic paraboloid.

Statement 59. A vector bundle whose base is contractible (for instance, a ball) is trivializable.

Statement 60. The euclidean orthogonal group has exactly two connected components.

4.2　False :

Statement 61. The stereographic projection of the sphere minus one point in the Euclidean space is bounded.

Statement 62. A holomorphic function on a Riemann surface is constant.

Statement 63. Any compact surface is diffeomorphic to an algebraic surface.

Statement 64. At any point P of a directrix of a hyperbola, two tangent lines intersect.

Statement 65. The orthogonal projection of the focus of a parabola on one of its tangent is on the directrix.

Statement 66. Any C^1 vector field on a torus admits a singularity.

4.3　Meaningless :

Statement 67. Any Riemannian metric is conjugated to the Haar measure.

Statement 68. The stereographic projection admits $\sqrt{2}$ as Euler characteristic.

Statement 69. The set of points equidistant from two Riemann surfaces is compatible with a paraboloid.

Statement 70. Any holomorphic compact fiber bundle is a particular sphere.

Statement 71. Any variety locally contractible is included in a two-sheeted hyperboloid.

Statement 72. Any locally ellipsoidal submersion is the exponential of a Riemann surface.

5　Non-math

5.1　True :

Statement 73. In all Ancient Mediterranean cultures, bulls were considered deities.

Statement 74. In Ancient Greece, a citizen who could not pay his debts was made a slave.

Statement 75. The VAT is a French invention and is a direct consumption tax.

Statement 76. The flag of the Esperanto community is predominantly green.

Statement 77. Apart from the Vatican, Gibraltar is the world's smallest country.

Statement 78. The concept of robots and avatars was already present in Greek mythology.

5.2 False :

Statement 79. The Paris metro was built before the Istanbul one.

Statement 80. All borders in Europe, except for Yugoslavia, were set at the end of World War II.

Statement 81. The poet Aragon never joined the Communist party.

Statement 82. The end of the Council of Trent coincides with the fall of the Western Roman Empire.

Statement 83. All members of the Club des Cordeliers were guillotined during the "Terror".

Statement 84. In every society, the market is considered an essential and founding institution.

5.3 Meaningless :

Statement 85. The potato flag was guillotined at the end of the Council of Trent.

Statement 86. The institutionalized market drinks Western Roman avatars.

Statement 87. Every indebted green beans have a scientific background.

Statement 88. The Greek mythology is the smallest alcohol derived from the VAT.

Statement 89. Most of the robotic bulls never met Yugoslavia.

Statement 90. A poet is a predominantly green tax over the metro.